

# Quarkonium in a weakly-coupled quark-gluon plasma

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**Abstract.** We report about a recent calculation of the heavy quarkonium mass and decay width in a quark-gluon plasma, whose temperature  $T$  and screening mass  $m_D$  satisfy the hierarchy  $m\alpha_s \gg \pi T \gg m\alpha_s^2 \gg m_D$ ,  $m$  being the heavy-quark mass, up to order  $m\alpha_s^5$ . The calculation may be relevant to understand the behavior of the  $\Upsilon(1S)$  in a quark-gluon plasma at present-day colliders.

**Keywords:** Quarkonium, Finite temperature, Spectrum, Decay

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## INTRODUCTION

The behavior of quarkonium in a medium has been the object of many studies since it was realized that the medium may induce quarkonium dissociation and that the dissociation pattern of the different quarkonia may provide an effective thermometer of the medium [1]. How the medium influences the quarkonium and eventually leads to its dissociation is the problem that such studies address.

In recent years, new insight in the problem has been gained by studying low-lying quarkonia in a weakly-coupled quark-gluon plasma. Low-lying quarkonia, like the bottomonium and charmonium ground states, are (believed to be) characterized by a typical inverse radius, which is of the order of  $m\alpha_s$ , larger than the non-perturbative scale of QCD,  $\Lambda_{\text{QCD}}$ , and a typical energy  $E$  of the quarks in the bound state, which is of the order of  $m\alpha_s^2$ , larger than or of the same order as  $\Lambda_{\text{QCD}}$ . Such states are Coulombic. In the case of the bottomonium ground state, it is often also assumed that  $E \gg \Lambda_{\text{QCD}}$ . In a weakly-coupled plasma, both the temperature  $T$  of the plasma and the screening mass,  $m_D$ , are larger than  $\Lambda_{\text{QCD}}$ . Moreover, the perturbative power counting implies that  $m_D$  is suppressed with respect to  $T$ , for  $m_D^2$  shows up at one loop. With these specifications, observables describing low-lying quarkonia in a weakly-coupled quark-gluon plasma may be calculated to a large extent analytically in perturbation theory, which makes the interest of these systems. As we will argue later, this could be the case of the  $\Upsilon(1S)$  at the LHC.

In [2, 3, 4, 5], the perturbative quarkonium static potential was studied for quark-antiquark distances  $r$  such that  $T \gg 1/r \gtrsim m_D$ . The surprising output of these studies has been that the imaginary part of the gluon self energy, which may be traced back to the Landau-damping phenomenon, induces an imaginary part of the static potential and hence a thermal width of the static quark-antiquark bound state. In [6], static particles in a hot QED plasma were considered in the situation  $1/r \sim m_D$ , confirming

previous results. In [7], the quarkonium static potential was first studied in an effective field theory (EFT) framework that exploits systematically the hierarchy of different energy scales in the problem. The static potential was studied for distances that range from larger to smaller than  $1/T$  and a new dissociation mechanism was identified. This is the color-singlet to color-octet break-up mechanism that provides the dominant contribution to the thermal decay width when  $E \gg m_D$ . In [8], moreover, the relation between the quarkonium potential and the correlation of two Polyakov loops, a quantity often evaluated on the lattice, has been investigated. A comprehensive study of non-relativistic bound states in a hot QED plasma in a non-relativistic EFT framework was performed in [9, 10]. It has been pointed out that quarkonium melts at temperatures larger than the melting temperature,  $T_{\text{melting}}$ , defined as the temperature for which the thermal decay width becomes as large as the binding energy; parametrically, it holds that  $\pi T_{\text{melting}} \sim mg^{4/3}$ . Since this temperature is lower than the temperature at which screening sets in, i.e. the temperature at which  $m_D$  is of the size of the inverse of the radius of the bound state, this finding challenges the long-time accepted view that quarkonium remains dissociated in a medium as long as color is screened; in fact, the existence of the melting temperature implies that quarkonium remains dissociated at lower temperatures, experiencing a larger dissociation rate.

In the following, we will report about the results of [11], where, in a specific range of temperatures, the spectrum and widths of quarkonia up to order  $m\alpha_s^5$  have been computed. In the assumed range of parameters, the scales of the bound state ( $m$ ,  $m\alpha_s$  and  $m\alpha_s^2$ ) and the thermodynamical scales ( $T$  and  $m_D$ ) fulfill a specific hierarchy that we will exploit either by constructing a corresponding hierarchy of EFTs or by subsequently integrating out different momentum regions.

## SCALES AND EFTS

We consider quarkonium in a thermal bath, whose energy scales satisfy the following hierarchy:

$$m \gg m\alpha_s \gg \pi T \gg m\alpha_s^2 \gg m_D. \quad (1)$$

This implies that  $mg^3 \gg T \gg mg^4$  and that  $\pi T$  is lower than  $\pi T_{\text{melting}}$ , i.e. quarkonium exists in the plasma. We will further assume that all these scales are larger than  $\Lambda_{\text{QCD}}$  and that a weak-coupling expansion is possible for all of them. Finally, in order to produce an expression for the spectrum that is accurate up to order  $m\alpha_s^5$ , we will assume that  $[m_D/(m\alpha_s^2)]^4 \ll g$ .

It is not clear if this hierarchy is realized by any of the quarkonium states at present heavy-ion facilities. Possible candidates are the bottomonium  $1S$  states ( $\Upsilon(1S)$ ,  $\eta_b$ ) at the LHC, for which it may hold  $m_b \approx 5 \text{ GeV} > m_b\alpha_s \approx 1.5 \text{ GeV} > \pi T \approx 1 \text{ GeV} > m\alpha_s^2 \approx 0.5 \text{ GeV} \gtrsim m_D$ .

It is useful to work in the so-called real-time formalism, which amounts to modifying the temporal integration in the partition function to include real times. As a consequence, the degrees of freedom double. However, the doubling of the degrees of freedom affects only gluon loops (at the order we are working only the gluon self energy), while physical heavy quarks decouple from unphysical ones [7], so the price to pay is minimal. The

advantage is that the real-time formalism allows a treatment of the quarkonium in the thermal bath very similar to the EFT framework developed for zero temperature [12]. Many results obtained there may, indeed, be simply translated here.

The EFT that follows from QCD by integrating out gluons and quarks of energy or momentum of order  $m$  in the quark-antiquark sector is non-relativistic QCD (NRQCD) [13]. The EFT that follows from NRQCD by integrating out gluons of energy or momentum of order  $m\alpha_s$  is potential non-relativistic QCD (pNRQCD) [14]. According to the hierarchy (1), both these scales are larger than  $T$ , which, therefore, may be set to zero in the matching to the EFT. As a consequence, the Lagrangians of NRQCD and pNRQCD are the same as at zero temperature.

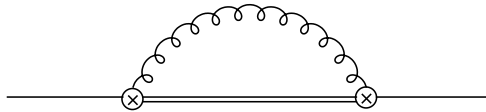
Integrating out  $T$  from pNRQCD modifies pNRQCD into hard-thermal loop (HTL) pNRQCD, pNRQCD<sub>HTL</sub>, [7, 15]. With respect to pNRQCD, the pNRQCD<sub>HTL</sub> Lagrangian gets relevant modifications in two parts. First, the Yang–Mills Lagrangian gets an additional HTL part [16]. This, for instance, modifies the longitudinal gluon propagator in Coulomb gauge into ( $k^2 \equiv \mathbf{k}^2$ )

$$\frac{i}{k^2} \rightarrow \frac{i}{k^2 + m_D^2 \left( 1 - \frac{k_0}{2k} \ln \frac{k_0 + k \pm i\eta}{k_0 - k \pm i\eta} \right)}, \quad (2)$$

where “+” identifies the retarded and “−” the advanced propagator. Second, the potentials get in addition to the Coulomb potential, which is the potential inherited from pNRQCD, a thermal part,  $\delta V$ .

## POTENTIAL, ENERGY AND DECAY WIDTH

In the following, we will provide the thermal corrections to the color-singlet quark-antiquark potential, the thermal corrections to the spectrum and the thermal decay width, aiming at a precision of the order of  $m\alpha_s^5$ .



**FIGURE 1.** The single line stands for a quark-antiquark color-singlet propagator, the double line for a quark-antiquark color-octet propagator and the circle with a cross for a chromoelectric dipole interaction.

### Integrating out the scale $T$

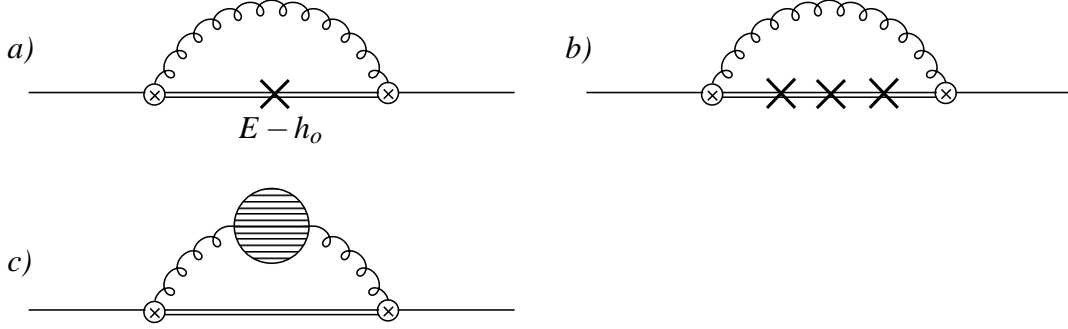
The relevant diagram contributing to the potential is shown in Fig. 1. It reads

$$-ig^2 \frac{4}{3} \frac{r^i}{D-1} \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{i}{E - h_o - k_0 + i\eta} [k_0^2 D_{ii}(k_0, k) + k^2 D_{00}(k_0, k)] r^i, \quad (3)$$

where  $D_{\mu\nu}$  stands for the gluon propagator,  $h_o = \mathbf{p}^2/m + \alpha_s/(6r)$  is the octet Hamiltonian and the loop integral has been regularized in dimensional regularization ( $D = 4 + \epsilon$

and  $\mu$  is the subtraction point). In the loop integral, we integrate over the momentum region  $k_0 \sim T$  and  $k \sim T$ . Since  $T \gg (E - h_o)$ , we may expand

$$\frac{i}{E - h_o - k_0 + i\eta} = \frac{i}{-k_0 + i\eta} - i \frac{E - h_o}{(-k_0 + i\eta)^2} + i \frac{(E - h_o)^2}{(-k_0 + i\eta)^3} - i \frac{(E - h_o)^3}{(-k_0 + i\eta)^4} + \dots$$



**FIGURE 2.** Detail of the different terms contributing to the potential.

The real part of the thermal correction to the color-singlet potential reads

$$\begin{aligned} \text{Re } \delta V_s(r) = & \frac{4\pi}{9} \alpha_s^2 r T^2 + \frac{8\pi}{9m} \alpha_s T^2 \\ & + \frac{4\alpha_s I_T}{3\pi} \left[ -\frac{9}{8} \frac{\alpha_s^3}{r} - \frac{17}{3} \frac{\alpha_s^2}{m r^2} + \frac{4}{9} \frac{\pi \alpha_s}{m^2} \delta^3(\mathbf{r}) + \frac{\alpha_s}{m^2} \left\{ \nabla_{\mathbf{r}}^2, \frac{1}{r} \right\} \right] \\ & - 2\zeta(3) \frac{\alpha_s}{\pi} r^2 T m_D^2 + \frac{8}{3} \zeta(3) \alpha_s^2 r^2 T^3, \end{aligned} \quad (4)$$

$$I_T = \frac{2}{\varepsilon} + \ln \frac{T^2}{\mu^2} - \gamma_E + \ln(4\pi) - \frac{5}{3}, \quad (5)$$

$$m_D^2 = g^2 T^2 \left( 1 + \frac{n_f}{6} \right), \quad (6)$$

where the first line of Eq. (4), which is of order  $g^2 r^2 T^3 \times E/T$ , comes from diagram *a*) in Fig. 2, the second one, which is of order  $g^2 r^2 T^3 \times (E/T)^3$ , comes from diagram *b*), and the third one, which is of order  $g^2 r^2 T^3 \times (m_D/T)^2$ , comes from diagram *c*);  $n_f$  is the number of light quarks.

The imaginary part of the color-singlet potential, which comes from diagram *c*) in Fig. 2, reads

$$\begin{aligned} \text{Im } \delta V_s(r) = & \frac{2}{9} \alpha_s r^2 T m_D^2 \left( -\frac{2}{\varepsilon} + \gamma_E + \ln \pi - \ln \frac{T^2}{\mu^2} + \frac{2}{3} - 4 \ln 2 - 2 \frac{\zeta'(2)}{\zeta(2)} \right) \\ & + \frac{16\pi}{9} \ln 2 \alpha_s^2 r^2 T^3. \end{aligned} \quad (7)$$

This contribution, which may be traced back to the Landau-damping phenomenon, is of order  $g^2 r^2 T^3 \times (m_D/T)^2$ .

Evaluating  $\text{Re } \delta V_s(r)$  and  $\text{Im } \delta V_s(r)$  on a quarkonium state with quantum numbers  $n$  and  $l$ , we obtain the thermal correction to the energy,  $\delta E_{n,l}^{(T)}$ , and the thermal width,

$\Gamma_{n,l}^{(T)}$ , coming from the scale  $T$ :

$$\begin{aligned} \delta E_{n,l}^{(T)} = & \frac{2\pi}{9} \alpha_s^2 T^2 a_0 [3n^2 - l(l+1)] + \frac{8\pi}{9m} \alpha_s T^2 \\ & + \frac{E_n I_T \alpha_s^3}{3\pi} \left\{ -\frac{32}{27} \frac{\delta_{l0}}{n} + \frac{200}{3} \frac{1}{n(2l+1)} - \frac{16}{3} \frac{1}{n^2} + \frac{27}{4} \right\} \\ & + \left( -\zeta(3) \frac{\alpha_s}{\pi} T m_D^2 + \frac{4}{3} \zeta(3) \alpha_s^2 T^3 \right) a_0^2 n^2 [5n^2 + 1 - 3l(l+1)], \quad (8) \end{aligned}$$

$$\begin{aligned} \Gamma_{n,l}^{(T)} = & \left[ -\frac{2}{9} \alpha_s T m_D^2 \left( -\frac{2}{\varepsilon} + \gamma_E + \ln \pi - \ln \frac{T^2}{\mu^2} + \frac{2}{3} - 4 \ln 2 - 2 \frac{\zeta'(2)}{\zeta(2)} \right) \right. \\ & \left. - \frac{16\pi}{9} \ln 2 \alpha_s^2 T^3 \right] a_0^2 n^2 [5n^2 + 1 - 3l(l+1)], \quad (9) \end{aligned}$$

where  $E_n = -\frac{1}{ma_0^2 n^2} = -\frac{4m\alpha_s^2}{9n^2}$  and  $a_0 = \frac{3}{2m\alpha_s}$ .

### Integrating out the scale $E$

The diagram shown in Fig. 1 also carries contributions coming from the energy scale  $E$ . They may be best evaluated in pNRQCD<sub>HTL</sub> by integrating over the momentum region  $k_0 \sim E$  and  $k \sim E$  and using HTL gluon propagators. Since  $k \sim E \ll T$ , we may expand the Bose–Einstein distribution

$$n_B(k) = \frac{T}{k} - \frac{1}{2} + \frac{k}{12T} + \dots; \quad (10)$$

moreover, since  $k \sim E \gg m_D$ , the HTL propagators can be expanded in  $m_D^2/E^2 \ll 1$ .

The momentum region  $k_0 \sim E$  and  $k \sim E$  is characterized by two possible momentum sub-regions. This can be understood by considering the integral

$$\int \frac{d^{D-1}k}{(2\pi)^{D-1}} \int_0^\infty dk_0 \frac{1}{2\pi} \frac{1}{k_0^2 - k^2 - m_D^2 + i\eta} \left( \frac{1}{E - h_o - k_0 + i\eta} + \frac{1}{E - h_o + k_0 + i\eta} \right). \quad (11)$$

For  $k_0 \sim E$  and  $k \sim E$ , it exhibits an off-shell sub-region,  $k_0 - k \sim E$ , and a collinear sub-region,  $k_0 - k \sim m_D^2/E$ . Note that, according to (1), the collinear scale satisfies  $mg^4 \gg m_D^2/E \gg mg^6$ , i.e. it is smaller than  $m_D$  by a factor of  $m_D/E \ll 1$  but still larger than the non-perturbative scale  $g^2 T$  by a factor  $T/E \gg 1$ .

The thermal correction to the energy,  $\delta E_{n,l}^{(E)}$ , coming from the scale  $E$ , reads

$$\delta E_{n,l}^{(E)} = -\frac{2\pi}{9} \alpha_s T m_D^2 a_0^2 n^2 [5n^2 + 1 - 3l(l+1)]. \quad (12)$$

We note the complete cancellation of the vacuum contribution (which includes the Bethe logarithm) against the thermal contribution originating from the “ $-1/2$ ” term in the expansion of the Bose–Einstein distribution (see Eq. (10)).

The thermal width,  $\Gamma_{n,l}^{(E)}$ , coming from the scale  $E$ , reads

$$\begin{aligned}
\Gamma_{n,l}^{(E)} = & 4\alpha_s^3 T - \frac{64}{9m} \alpha_s T E_n + \frac{32}{3} \alpha_s^2 T \frac{1}{mn^2 a_0} \\
& + \frac{2E_n \alpha_s^3}{3} \left\{ -\frac{32}{27} \frac{\delta_{l0}}{n} + \frac{200}{3} \frac{1}{n(2l+1)} - \frac{16}{3} \frac{1}{n^2} + \frac{27}{4} \right\} \\
& - \frac{2}{9} \alpha_s T m_D^2 \left( \frac{2}{\varepsilon} + \ln \frac{E_1^2}{\mu^2} + \gamma_E - \frac{11}{3} - \ln \pi + \ln 4 \right) a_0^2 n^2 [5n^2 + 1 - 3l(l+1)] \\
& + \frac{128 T m_D^2}{81} \frac{\alpha_s^3}{E_n^2} I_{n,l}, \tag{13}
\end{aligned}$$

where  $I_{1,0} = -0.49673$ ,  $I_{2,0} = 0.64070$ ,  $\dots$ . The leading contribution is given by the first three terms, which are of order  $\alpha_s^3 T$ . This contribution to the thermal width is generated by the possible break up of a quark-antiquark color-singlet state into an unbound quark-antiquark color-octet state: a process that is kinematically allowed only in a medium. The singlet to octet break up is a different phenomenon with respect to the Landau damping. In the situation  $E \gg m_D$ , the first dominates over the second by a factor  $(m\alpha_s^2/m_D)^2$ .

### Integrating out the scale $m_D$

The diagram shown in Fig. 1 also carries contributions coming from the energy scale  $m_D$ . These contributions are suppressed with respect to the other terms calculated.


## CANCELLATION OF DIVERGENCES

The thermal corrections to the spectrum and the thermal decay width develop divergences at the different energy scales. These are artifacts of the scale separations and cancel in the final (physical) results.

Concerning the thermal decay width, the divergence at the scale  $m\alpha_s^2$  in Eq. (13), which is of ultraviolet (UV) origin, cancels against the infrared (IR) divergence at the scale  $T$  in Eq. (9).

In the spectrum, the pattern of divergences is more complicated and is summarized in Tab. 1. The table may be read vertically or horizontally. If read vertically, it shows a typical EFT cancellation mechanism: at the scale  $m\alpha_s$  we have non-thermal IR divergences in the potentials, these cancel against non-thermal UV divergences at the scale  $m\alpha_s^2$  [17], the non-thermal contribution at the scale  $T$  is scaleless and vanishes in dimensional regularization; thermal IR divergences at the scale  $T$  cancel against thermal UV divergences at the scale  $m\alpha_s^2$ . If read horizontally, it shows a cancellation mechanism that is familiar in thermal field theory: at the scale  $T$ , thermal IR divergences cancel against non-thermal IR divergences while non-thermal UV divergences cancel against IR divergences that appear in the potentials at the scale  $m\alpha_s$ ; at the scale  $m\alpha_s^2$  UV thermal divergences cancel against UV non-thermal divergences. The cancellation between

**TABLE 1.** The pattern of IR and UV divergences in the quarkonium spectrum at different energy scales. The final result is finite.

Scale	Vacuum	Thermal
$m\alpha_s$	$\sim m\alpha_s^5 \frac{1}{\epsilon_{\text{IR}}}$	
$T$	$\sim m\alpha_s^5 \left( \frac{1}{\epsilon_{\text{IR}}} - \frac{1}{\epsilon_{\text{UV}}} \right)$	$\sim -m\alpha_s^5 \frac{1}{\epsilon_{\text{IR}}}$
$m\alpha_s^2$	$\sim -m\alpha_s^5 \frac{1}{\epsilon_{\text{UV}}}$	$\sim m\alpha_s^5 \frac{1}{\epsilon_{\text{UV}}}$

non-thermal and thermal parts is possible because the latter may carry temperature independent terms (see, for instance, the “ $-1/2$ ” term in Eq. (10)). Note that both at the scales  $T$  and  $m\alpha_s^2$ , the spectrum is finite.

## SUMMARY

For a quarkonium state that satisfies the hierarchy specified in Eq. (1) and in the following discussion, the complete thermal contribution to the spectrum up to  $\mathcal{O}(m\alpha_s^5)$  is obtained by summing Eqs. (8) and (12) and subtracting from the latter the zero-temperature part, this gives

$$\begin{aligned}
\delta E_{n,l}^{(\text{thermal})} = & \frac{2\pi}{9} \alpha_s^2 T^2 a_0 \left[ 3n^2 - l(l+1) + \frac{8}{3} \right] \\
& + \frac{E_n \alpha_s^3}{3\pi} \left[ \log \left( \frac{2\pi T}{E_1} \right)^2 - 2\gamma_E \right] \\
& \times \left\{ -\frac{32}{27} \frac{\delta_{l0}}{n} + \frac{200}{3} \frac{1}{n(2l+1)} - \frac{16}{3} \frac{1}{n^2} + \frac{27}{4} \right\} \\
& + \frac{128 E_n \alpha_s^3}{81\pi} L_{n,l} \\
& + a_0^2 n^2 [5n^2 + 1 - 3l(l+1)] \left\{ - \left[ \frac{1}{\pi} \zeta(3) + \frac{2\pi}{9} \right] \alpha_s T m_D^2 \right. \\
& \left. + \frac{4}{3} \zeta(3) \alpha_s^2 T^3 \right\}, \tag{14}
\end{aligned}$$

where  $L_{n,l}$  are the QCD Bethe logarithms:  $L_{1,0} = -81.5379$ ,  $L_{2,0} = -37.6710$ , ... [18].

For a quarkonium state that satisfies the hierarchy specified in Eq. (1) and in the following discussion, the complete thermal width up to  $\mathcal{O}(m\alpha_s^5)$  is obtained by summing

Eqs. (9) and (13), this gives

$$\begin{aligned}
\Gamma_{n,l}^{(\text{thermal})} = & \left(4 + \frac{832}{81} \frac{1}{n^2}\right) \alpha_s^3 T \\
& + \frac{2E_n \alpha_s^3}{3} \left\{ -\frac{32}{27} \frac{\delta_{l0}}{n} + \frac{200}{3} \frac{1}{n(2l+1)} - \frac{16}{3} \frac{1}{n^2} + \frac{27}{4} \right\} \\
& - \left[ \frac{2}{9} \alpha_s T m_D^2 \left( \ln \frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2 \frac{\zeta'(2)}{\zeta(2)} \right) + \frac{16\pi}{9} \ln 2 \alpha_s^2 T^3 \right] \\
& \times a_0^2 n^2 [5n^2 + 1 - 3l(l+1)] \\
& + \frac{32}{9} \alpha_s T m_D^2 a_0^2 n^4 I_{n,l}.
\end{aligned} \tag{15}$$

As a qualitative summary, we observe that, at leading order, the quarkonium masses increase quadratically with  $T$ , which implies the same functional increase in the energy of the leptons and photons produced in the electromagnetic decays. Electromagnetic decays occur at short distances  $\sim 1/m \ll 1/T$ , hence the standard NRQCD factorization formulas hold. At leading order, all the temperature dependence is encoded in the wave function at the origin. The leading temperature correction to it can be read from the potential and is of order  $n^4 T^2 / (m^2 \alpha_s)$ . Hence, a quadratic dependence on the temperature should be observed in the frequency of produced leptons or photons. Finally, at leading order, a decay width linear with temperature is developed. The mechanism underlying this decay width is the color-singlet to color-octet thermal break-up, which implies a tendency of the quarkonium to decay into a continuum of color-octet states.

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